XV. POSITIONING AUTOMATED GUIDED VEHICLES IN A GENERAL GUIDE-PATH LAYOUT

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Abstract

The locations of dwell points for idle vehicles in an automated guided vehicle (AGV) system determine the response times for pick-up requests and thus affect the performance of automated manufacturing systems. In this study, we address the problem of optimally locating dwell points for multiple AGVs in general guide-path layouts with the objective of minimizing the maximum response time in the system. We propose a mixed integer linear programming (MILP) formulation for the problem. We also develop a generic genetic algorithm (GA) to find near optimal solutions. The MILP model and GA procedure are illustrated using a two-dimensional grid layout problem. Our computational study shows that the proposed GA procedure can yield near optimal solutions for these test problems in reasonable time.

Keywords: Automated Guided Vehicle System; Dwell Point Location; Mixed Integer Linear Programming Model; Genetic Algorithm.

1 Introduction

An AGV is a driverless vehicle that follows wires in the floor, or uses vision or lasers. AGV systems are most often used in industrial applications to move materials around a manufacturing facility or a warehouse. They have been implemented in a large variety of industries such as aerospace, automotive, chemical, electronics, plastic, food and

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beverage, and textiles as well as in inter-modal container ports. AGV system implementations could potentially lead to better production planning and control, safety, cost reduction, and flexibility. AGVs also offer a seamless interface with increasingly used automated warehousing systems, which can be controlled by integrated warehousing management techniques. However, these potential benefits can only be obtained when adequate guide-path layouts and control systems for vehicle dispatching, routing, and traffic management are available (Ventura and Lee [7]).

Logistics problems in the implementation of an AGV system comprise location of pick-up and delivery (P/D) points, optimal guide-path design, determining the optimal number and type of AGVs, positioning of AGVs, assignment of AVGs to pick-up requests, routing and dispatching of AGVs, and resolution of deadlocks and routing conflicts. Among these problems, positioning of AGVs and assignment to pick-up requests is an important control issue in AGV system implementation (Asef-Vaziri and Laporte [1]). Since the workload of a manufacturing system changes over time, the idleness of the material handling equipment used in the facility is expected in order to avoid system overload. Thus, a relevant problem in an AGV system is to decide the location of an AGV when it finishes a delivery job and has no immediate assignments, because the position of the idle vehicle determines the empty travel time to the next pick-up request, which is called the response time. The reduction of response times contributes to the reduction of the overall material handling time and this is an important objective in the material handling design process. In particular, the objective of minimizing the maximum response time is important in just-in-time manufacturing facilities and distribution centers, where the emphasis is to reduce large turnaround times (Ventura and Lee [7]). In certain multi-product multi-line assembly systems using AGV systems, the parts required for a particular assembly operation are transported by AGVs from storage bins to kitting stations to form kits. The time required to form a complete kit depends on the maximum of the travel times of the AGVs that transport the required parts. In distribution centers, items related to a single customer order must be collected for packing and delivery. Here, the maximum response time determines the overall turnaround time. Reduction of turnaround times is crucial as the order picking comprises as much as 65% of the operating cost of a typical distribution center (Coyle et al. [2]).

Egbelu [3] considered the problem of locating dwell points for AGVs to minimize the maximum response time in a single loop guide-path layout. He provided optimal solutions for the location of a single AGV in uni-directional and bi-directional guide-paths and proposed heuristic algorithms for the location of multi-vehicles in both types of guide-paths. He illustrated the algorithms using small-size example problems. Gademann and Van de Velde [5] considered the uni-directional single loop guide-path layout and proposed a dynamic programming (DP) algorithm to minimize any regular function of the response times. The proposed DP algorithm has a complexity of $O(nm^3)$, where m is the number of pickup points and n is the number of AGVs, in minimizing an arbitrary regular function of the response times, and $O(m^2 \log m)$ in minimizing the maximum

response time. Ventura and Lee [7] considered the uni-directional single loop guide-path layout and addressed the problem of optimally locating multiple dwell points for the AGVs with the objective of minimizing the maximum response time. A DP algorithm with $O\left(n(m-n)^2\right)$ complexity was developed to optimally solve the problem. The authors showed that the deviation between the solution provided by Egbelu's heuristic algorithm and the optimal solution could be significantly high. Ventura and Rieksts [8] developed a DP algorithm to solve idle vehicle positioning problems in uni-directional single loop systems to minimize the maximum response time considering restrictions on vehicle time availability. They also determined the minimum number of vehicles to handle the workload for pickup requests.

In recent years, general guide-path layouts with cross-aisle and angled-aisle structure, which are special cases of grid type layouts, are becoming more prevalent in manufacturing industries and warehouses in order to utilize space more efficiently and to reduce travel times between locations. Examples of guide-path layout topologies include the circle or loop layout, tandem loop layout, uni-dimensional grid layout, and two-dimensional grid layout (see Figure 1).

Circle or loop layout

Tandem loop layout

Uni-dimensional grid layout

Two-dimensional grid layout

○ - P/D Station   □ - Intersection   ▦ - Transfer

Figure 1: Examples of general guide-path layout topologies for AGV systems

To our knowledge, there is no publication that considers locating idle AGVs in general guide-path layouts. This study addresses the issue of optimally determining dwell points for multiple AGVs in general guide-path layouts, which can be modeled as directed networks. We show that placing dwell points either at the P/D stations or intersection points with out-degree of at least two minimizes the maximum response time in general guide-path layouts. We also show that the intersection points with out-degree equal to 1 do not qualify as potential dwell point locations. We propose a mixed-integer linear programming (MILP) model for the solving the problem with the objective of minimizing the maximum response time in the system. We also propose a genetic algorithm (GA) which can find near optimal solutions. The MILP model and the GA procedure are illustrated using an example of a two-dimensional grid layout.

The remainder of this paper is organized as follows. In Section 2, we describe the dwell point location problem and develop an MILP model for the problem. In Section 3, we propose a generic GA to solve the problem for minimizing any linear/non-linear function of the response times. In Section 4, we present an illustrative example to show the application of the proposed mathematical model and GA procedure. Section 5 presents some conclusions on the work.

2   Problem description and mathematical model

In this section, we present an MILP model to optimally locate dwell points for AGVs in general guide-path layouts, which can be characterized as directed networks. In a uni-dimensional grid layout, all AGVs move in the same direction, either clockwise or counter-clockwise, while in a two-dimensional grid layout, vehicles can change direction depending on the arc they are traversing. The underlying topology of an AGV guide-path can be modeled as a general directed network $G=(V, A)$, where V is the set of nodes and A comprises the arcs defined by pairs of nodes in V. An intersection point can be defined as a point where at least three guide-path segments (arcs) meet, which may or may not coincide with a pick-up and drop-off (P/D) station. Note that, while circular or loop layouts do not have intersection points, uni-dimensional (non-circular) and two-dimensional grid type layouts must have at least one. A node of G may be a P/D station, an intersection point, or an intersection point with a P/D station. Let $I=\{v_1,v_2,...,v_m\}$ be the set comprising the m P/D stations. Let $Q= \{y_1, y_2, ...,y_q\}$ be the set containing the q intersection points in the guide-path. Then, $V = I \cup Q$. Note that, in well-designed guide-path systems, the out-degree of all nodes of G must be at least 1. Otherwise, a node with an out-degree of 0 would create a deadlock state.

Let $J = \{x_1, x_2, ...,x_n\}$ be a set of dwell points in G where the n AGVs are positioned when they become idle. In addition, let $S = \{S_1, S_2, ...,S_n\}$ be a partition of set I, where S_j includes the P/D stations served by the AGV assigned to dwell point x_j. Note that a pair (J, S) provides a feasible solution to the dwell point location problem as long

as the dwell points in J belong to G, S is a partition of I, and vehicle time restrictions are satisfied.

Let $d(x_j, v_i)$ be the shortest distance from dwell point x_j to pick-up station v_i, for $x_j \in J$, $v_i \in I$. It is assumed that $d(x_j, v_i) = 0$ if and only if $v_i = x_j$; otherwise, $d(x_j, v_i) > 0$; We use the Floyd–Warshall algorithm ([6]) to determine the lengths of the shortest paths between dwell points and P/D stations. Let s_E be the speed of a vehicle when it travels empty and $r_{ji} = d(x_j, v_i)/s_E$ be the response time from dwell point x_j to pick-up station v_i. The response time r_{ji} for a request from P/D station v_i handled by an AGV located in dwell point x_j is the empty travel time from x_j to v_i. We partition the set of intersection points in Q into Q' and Q'', where Q' is the set of intersection points in G with out-degree equal to 1 and Q'' is the set of intersection nodes in G with out-degree of at least 2. Thus, $Q = Q' \cup Q''$ and $V = I \cup Q' \cup Q''$. In addition, let $f_r(J, S)$ be the objective function for the dwell point location problem representing a regular performance measure of response times. Thus, $f_r(J, S)$ is a non-decreasing function of the response times $\{r_{ji} : x_j \in J, v_i \in S_j\}$.

Ventura and Rieksts [8] proved that there exists an optimal set of dwell points that minimizes the maximum response time in a loop guide-path layout, where all dwell points coincide with P/D station locations. Ventura et al. [9] showed that, for any regular performance measure of response times $f_r(J, S)$ for an AGV system with n vehicles and m P/D stations in a general guide-path layout, there exists an optimal solution (J^*, S^*), where dwell points are either P/D station locations or intersection points with out-degree of at least 2, i.e., $J^* \subseteq I \cup Q''$ (see Theorem 1 in [9]) .

Problem (P): Minimizing the maximum response time ([3], [7])
The objective of the problem is to optimally determine dwell points for idle AGVs that minimizes the maximum response time in a general guide-path layout. Let R be the maximum response time of the system that needs to be minimized.

Based on Theorem 1 in [9], $I \cup Q'' = \{x_1, x_2, \ldots, x_p\}$ is the set of potential dwell points for positioning idle AGVs, where $p = m + q''$ is the cardinality of $I \cup Q''$ and q'' is the number of intersection points with out-degree of at least 2.

The proposed MILP model uses the following decision variables:

$$X_{ij} = \begin{cases} 1, & \text{if station } v_i \text{ is assigned to the AGV in dwell point } x_j, \\ 0, & \text{otherwise,} \end{cases} \quad \text{for } i = 1, 2, \ldots, m, j = 1, 2, \ldots, p.$$

$$\delta_j = \begin{cases} 1, & \text{if an AGV is located in dwell point } x_j, \\ 0, & \text{otherwise,} \end{cases} \quad \text{for } j = 1, 2, \ldots, p.$$

The assumptions considered in the model are given below:
  a) We assume that $|I| > n$.

b) The velocity of the vehicles is assumed to be constant.

c) Traffic interference between vehicles is not taken into consideration.

d) The loading time at the pick-up station, transport time to the drop-off station, unloading time at the drop-off station, and the time to return to the dwell point are insignificant.

e) It is assumed that an AGV can serve all the pick-up requests assigned to it.

f) All the pick-up requests from a station are served by the same AGV.

The following MILP model determines the optimal response time and set of dwell points that minimize the maximum response time:

(P)   Minimize R,

       subject to

$$\sum_{j=1}^{p} X_{ij} = 1, \qquad \forall i = 1, 2, ..., m, \tag{1}$$

$$\sum_{i=1}^{m} X_{ij} \leq |I|\, \delta_j, \qquad \forall j = 1, 2, ..., p, \tag{2}$$

$$\sum_{j=1}^{p} \delta_j = n, \tag{3}$$

$$r_{ji}\, X_{ij} \leq R, \qquad \forall i = 1, 2, ..., m, \forall j = 1, 2, ..., p, \tag{4}$$

$$X_{ij} \in \{0,1\}, \qquad \forall i = 1, 2, ..., m, \forall j = 1, 2, ..., p, \tag{5}$$

$$\delta_j \in \{0,1\}, \qquad \forall j = 1, 2, ..., p, \tag{6}$$

$$R \geq 0. \tag{7}$$

In this model, constraint set (1) ensures that each P/D station is assigned to a single AGV prepositioned at a precise dwell point. Constraint set (2) ensures that P/D stations are assigned to a potential dwell point x_j only when an AGV is assigned to x_j ($\delta_j = 1$). Constraint (3) ensures that exactly n dwell points are selected. Constraint set (4) is used to determine the maximum of the response times among the pick-up requests from all the P/D stations. Constraint sets (5) and (6) show the binary variables considered in the model. Constraint (7) is the non-negativity constraint for the maximum response time variable.

3 Genetic algorithm (GA) for locating dwell points in general guide-path layouts

Large-scale NP-hard problems are difficult to solve using mathematical models in a general computational sense. The time complexity of the problem increases exponentially as a function of the problem size. GAs are considered to be a powerful set of global search techniques that have been shown to produce very good results for a wide class of

NP-hard problems (Paul and Rajendran [6]). In this study, we propose a generic GA procedure for the dwell point location problem for general guide-path layouts. The GA procedure can be easily adapted to solve the problem with any linear/non-linear function of the response times by modifying the fitness function appropriately. The terminology and steps of the GA procedure are provided below.

Notation

f_c	:	Fitness value of chromosome c.
RP_c	:	Relative fitness of chromosome c in the parent population.
CP_c	:	Cumulative fitness of chromosome c in the parent population.
L	:	Length of the chromosome.
max_gen	:	Maximum number of generations (termination criteria).
MR	:	Mutation rate.
N	:	Population size.
no_gen	:	Number of the current generation.
par_pop	:	Chromosomes in the parent population.
off_pop	:	Chromosomes in the offspring population.
u	:	Uniform random number between 0 and 1.
Z(c)	:	Objective function value of chromosome c.

Chromosome representation

In this study, a feasible solution to the dwell point location problem can be represented in vector form. For a problem with p potential dwell points, the length of the chromosome (L) is equal to p. For example, consider a loop layout with 7 stations and 3 AGVs (*i.e.*, 7 stations represent the potential dwell point locations). The permutation of potential dwell point indexes represents the chromosome. The chromosome representation is shown in Figure 2. In this example, the location of the dwell points would be in stations 1, 3, and 5.
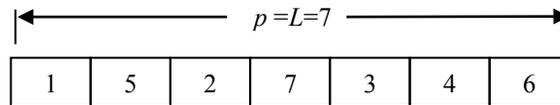
$$p = L = 7$$

1	5	2	7	3	4	6

Figure 2: Chromosome representation

Initial population

Population size is one of the GA parameters that can significantly affect its performance. Diversity in population is one of the key requirements for quick convergence of any GA methodology ([6]). Based on initial experimentation, we choose to have a population size (N) of 30. The chromosomes in the initial population are generated randomly.

Once the locations of the dwell points are known, the assignment of dwell points to the P/D stations is done based on the least response time.

Fitness value

As the objective function o this problem has to be minimized, the fitness value of each chromosome is calculated by the following fitness function $(f_c) = \dfrac{1}{1+Z(c)}$.

Fitness function for problem $(P) = \dfrac{1}{(1+R)}$, where $R = \max \{r_{ji}\, X_{ij} : \forall i = 1,...,m,\ \forall j = 1,...,p\}$.

The step by step procedure for the proposed GA is presented in this section.

STEP 1: Initialize $no\_gen = 0$.

STEP 2: Generate N chromosomes randomly for the initial population. Each gene represents a potential dwell point index and each chromosome represents a permutation of potential dwell point positions.

STEP 3: Evaluate the fitness function (f_c) of the chromosome in the initial population.

STEP 4: With the fitness value f_c, calculate the selection probability RP_c and cumulative probability CP_c for every chromosome c in par_pop.

STEP 5: Perform the crossover operation from par_pop, to obtain N offspring chromosomes. Set these offspring chromosomes to off_pop:
In the GA-GX, off_pop is generated by using gene-wise crossover (GX) mechanism [9].
In the GA-OX, off_pop is generated by using order crossover (OX) mechanism [9].
In the GA- GX-OX, off_pop is generated is using hybrid crossover mechanism [9]. Generate u. If $0 \leq u \leq 0.5$, GX crossover is used to generate off_pop; If $u > 0.5$, OX crossover is used to generate off_pop.
Detailed implementation steps of the GX and OX mechanisms are given in [9].

STEP 6: The N chromosomes in off_pop is subjected to mutation, with a probability of MR. We use the swap mutation operator in this study [9].

STEP 7: Evaluate the fitness function (f_c) of every chromosome in off_pop.

STEP 8: From both par_pop and off_pop, select the best N distinct chromosomes based on the fitness value, to form the par_pop for the next generation.

STEP 9: Increment $no\_gen = no\_gen+1$;
If $no\_gen < max\_gen$, then return to STEP 4; else proceed to STEP 10.

STEP 10: Stop. The best solution (*i.e.*, the best chromosome) among the chromosomes in the final par_pop and its objective function value constitute the best known solution to the problem.

A detailed discussion on the crossover mechanisms and parameter settings of the GA are given in [9].

4 Illustrative example

In this section, we illustrate the application of the proposed MILP model and GA based methodology (with three types of crossover mechanisms) to a two-dimensional grid layout problem. We consider the objective of minimizing the maximum response time. Consider the grid layout example in Figure 3. The number of stations (m) is 15. In this example, it is assumed that $s_E = s_B = 1$. There are seven intersections in the layout of which only two intersections (which are numbered as 16 and 17) have an out-degree 2. The other five intersections, with an out degree 1, do not qualify as potential dwell points (based on Theorem 1 in [9]). So, the number of potential dwell points (p) is 17 (15 P/D stations and 2 intersections).The number of AGVs (n) in the system is taken as 3. The distances between the stations and potential dwell points are given in Table 1.
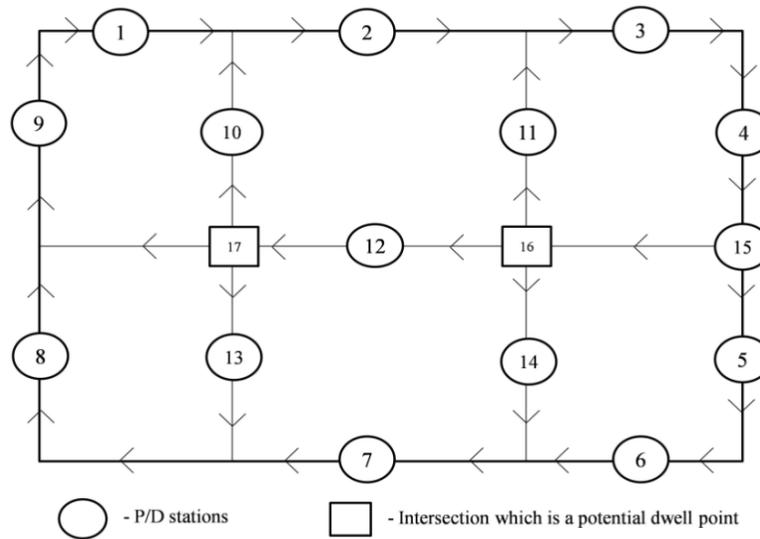

Figure 3: Two-dimensional grid layout (illustrative example) [9]

Table 1: Distance between the stations (illustrative example: two-dimensional grid layout)

From P/D station	To P/D station	Distance	From P/D station	To P/D station	Distance
1	2	1.94251	12	17	3.27245
2	3	5.21129	13	8	3.1768
3	4	1.67016	14	7	7.15363
4	15	2.54193	15	5	4.68419
5	6	5.56047	15	16	7.02973
6	7	8.5693	16	11	2.38905
7	8	5.49253	16	12	3.12622
8	9	1.97007	16	14	4.03392

9	1	2.37270	17	9	4.32003
10	2	2.44572	17	10	8.68274
11	3	9.06542	17	13	5.85618

We solve the illustrative example using the MILP model and the three versions of the GA for problem (*P*). The results are shown in Table 2.

Table 2: Results of the illustrative example: two-dimensional grid layout

	Optimal solution (MILP)	CPU time (MILP) in seconds	GA objective			Best GA objective	CPU time (GA) in seconds	Best solution: iteration number in GA	Best solution CPU time in GA (in seconds)	% deviation of GA solution from optimal
			GX	OX	GX-OX					
(*P*)	12.7866	2.94	12.7866	12.7866	12.7866	12.7866	< 0.01	10	< 0.01	0.00%

GA is able to find the optimal solution for this illustrative example for problem (*P*). GA provides an alternate optimal solution for problem. The dwell point locations and the assignment of the stations to the dwell points are shown in Table 3.

Table 3: Dwell point locations and their assignment (illustrative example: two-dimensional grid layout)

		Dwell point	Stations assigned to the dwell point
(*P*)	Optimal solution	Station 4	4, 5, 6, 11, 15
		Station 12	1, 2, 8, 10, 13
		Intersection16	3, 7, 9, 12, 14
	GA solution	Station 4	4, 5, 6, 15
		Station 12	1, 2, 8, 9, 10, 12, 13
		Intersection16	3, 7, 11, 14

The proposed GA procedure yields an optimal solution for the two-dimensional grid layout problem. A computational study has also been performed on loop layout and two cases of two-dimensional grid networks in [9] and the results are promising. The average deviation of GA solution from optimal is 1.75% and less than 2% for loop layout and two-dimensional grid network problems respectively ([9]).

5 Conclusions

AGV systems provide a promising material handling solution which can improve productivity of automated manufacturing systems and distribution centers along with reducing labor costs, material handling damage, and increasing dependability and safety. General directed guide-path layouts, such as grid type layouts, are becoming more common in manufacturing and warehouse systems for efficient usage of storage space and reduction of travel times between storage locations. In this study, we have considered the problem of locating dwell points for idle vehicles in an AGV system. We have

addressed the problem of optimally determining dwell points for multiple AGVs in general directed networks with the objective of minimizing the maximum response time. We have developed an MILP model and a generic GA procedure for solving the problem. We have illustrated the model and the GA procedure using a two-dimensional grid layout example. The proposed GA was able to give an optimal solution for the illustrative example. Our computational study on the loop layout and two-dimensional grid network problems showed that the GA procedure was able to find near optimal solutions in reasonable CPU times. This proves the potential of the proposed GA to handle real-time problems.

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